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Stationary problem of a simple chemotaxis-growth model*

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1 Introduction

Mathematical model for pattern dynamics of aggregating region of biological individuals possessing the chemotaxis was proposed in [3], [19], [21], [25] as follows:

$$\begin{cases} u_t = \mathcal{D}\nabla\{\nabla u - \alpha u \nabla\chi(v)\} + f(u), & (x,t) \in \Omega\times\mathbf{R}_+, \\ v_t = d\Delta v + u - v, & (x,t) \in \Omega\times\mathbf{R}_+, \\ u(\cdot,0) = u_0 \geq 0, \quad v(\cdot,0) = v_0 \geq 0, & x \in \Omega, \\ u_\nu(x,\ t) = 0,\ v_\nu(x,\ t) = 0, & (x,t) \in \partial\Omega\times\mathbf{R}_+, \end{cases} \tag{1}$$

where $\mathcal{D}$, d and α are positive constants and $\Omega \subset \mathbf{R}^N$ $(N \leq 3)$ denotes a bounded domain with smooth boundary $\partial\Omega$. The sensitive function $\chi(v)$ satisfies $\chi'(v) > 0$ for $0 < v$. In this paper, we treat the logistic growth term $f(u)$ given by

$$f(u) = u(1-u).$$

For this model, several spatio-temporal patterns due to the Türing and Hopf instability induced the chemotaxis have been investigated by many people ([2], [13], [14], [26], [31]). Specifically, this model exhibits that there are many static and dynamic patterns in virtue of the balance between three effects, chemotaxis, diffusion and growth. In the critical case, as chemotaxis effect is very strong, static and chaotic spots patterns is introduced in [2], [10], [26] as $N = 1, 2$. It is one of the features which the system of Keller-Segel type [12] exhibits. On the other hand, Yagi et al. [2] show that the existence of the global solutions of (1) and the exponential attractor, which dimension is growing as $\alpha \to \infty$. This result implies that the dynamics induced from (1) becomes more complex under this situation.

Here, we only study the stationary problem of (1) as follows:

$$\begin{cases} \mathcal{D}\nabla\{\nabla u - \alpha u \nabla\chi(v)\} + f(u) = 0, & x \in \Omega, \\ d\Delta v + u - v = 0, & x \in \Omega, \\ u \geq 0,\ v \geq 0, & x \in \Omega \\ u_\nu(x) = v_\nu(x) = 0, & x \in \partial\Omega. \end{cases} \tag{2}$$

For suitable values of parameters $\mathcal{D}$, α and d, the existence and nonexistence of the stationary solution are studied in [6], [13], [14], [31]. Therefore, our goal is to obtain all solutions of (2).

*This is a joint work with Kousuke Kuto and Yasuhito Miyamoto

Recently, the global structure of the stationary solution bifurcated from the constant solution for one dimensional domain is shown by Gai et al. [7]. On the other hand, there are static and moving spots patterns by several numerical simulations ([2], [10]) in the case which the chemotaxis effect is very strong, that is, $\alpha \gg 1$. Another motivation is to show the existence of the spiky solution which corresponds to a concentrative pattern. To do so, we first treat the case $\alpha \to \infty$. Then, the solution of (2) converges to each one of two constant solutions $(0,0)$, $(1,1)$ with respect to suitable norm ([7], [14]). Therefore, the solution set of (2) is not so complex in the stationary problem as α is very large. If there exists a sequence of the solutions which converges to (0, 0), these solution has several peak points because of $\min_{\overline{\Omega}} u \le 1 \le \max_{\overline{\Omega}} u$ (see [7], [14]). We think that these sequence corresponds to the spots pattern obtained by the numerical simulations, but the existence is not proved.

On the other hand, we consider the case when one of the diffusion coefficients and chemotaxis intensity are both large. We treated the same situation for the other model and obtain several results about the stationary problem ([15], [16], [17]). Our interest is to show the global structure of the stationary solution of (2) depending on the parameter α as $\mathcal{D} \to \infty$. Then, we formally obtain the limiting system as follows:

$$\begin{cases} \nabla\{\nabla u - \alpha u \nabla \chi(v)\} = 0, & x \in \Omega, \\ d\Delta v + u - v = 0, & x \in \Omega, \\ u \ge 0, \ v \ge 0, & x \in \Omega, \\ u_\nu(x) = v_\nu(x) = 0, & x \in \partial\Omega \end{cases} \tag{3}$$

and

$$\int_\Omega f(u)\, dx = 0. \tag{4}$$

Since (3) becomes the stationary problem of Keller-Segel type system, there are many results of the solutions of this system (e. g. [4], [11], [20], [27], [29]).

The organization of this paper is as follows: In Section 2, it is proved that there exists a sequence of the solutions of (2) which converge to the solution of (3), (4) as $\mathcal{D} \to \infty$. For $\Omega = (0,1)$, we show the global structure of solutions of (3), (4) depending on the the parameters d, α in Section 3. In these two sections, we treat the simple sensitive function $\chi(v) = v$. In Section 4, we similarly consider the existence of the solution of (3), (4) with $\chi(v) = \log v$.

2 Convergence theorem for stationary solutions as $N = 2,\ 3$

In this section, we consider the convergence property for solutions of (2) with $\chi(v) = v$ as $\mathcal{D}$ tends to ∞. First we will show the universal bound for the solutions of (2) with respect to $\mathcal{D}$ and d. Using $f(u) = u(1-u)$ and applying the elliptic regularity theory to (2), we can easily prove

Lemma 2.1 *There is a constant C depending on $\partial\Omega$ such that*

$$\|u\|_{L^1} = \|u\|_{L^2} \le |\Omega| \tag{5}$$

and

$$\|v\|_{L^1} \le |\Omega|, \quad \|v\|_{H^2} \le \frac{C}{d}\|u\|_{L^2}. \tag{6}$$

Lemma 2.2 *For any positive constant A, there exists a positive constant C independent of $\mathcal{D}$, d such that for any $A < d, \mathcal{D}$*

$$\|u\|_{W^{2,6}(\Omega)} < C, \ \|v\|_{W^{2,6}(\Omega)} < C \tag{7}$$

for any positive solutions (u, v) of (2).

Proof. Integrating the first equation of (2) and using the boundary conditions, we have

$$\int_\Omega |\nabla u|^2 \le \frac{1}{\mathcal{D}}\left\{\|u\|_{L^1}+\|u\|_{L^2}^2\right\}+\alpha\int_\Omega u\nabla u\nabla v \le \frac{1}{\mathcal{D}}\left\{\|u\|_{L^1}+\|u\|_{L^2}^2\right\}+\frac{\alpha}{2}\int_\Omega \nabla u^2\nabla v. \quad (8)$$

It follows from the Gagliardo-Nirenberg inequality (see [1]) that

$$\int_\Omega \nabla u^2\nabla v = -\int_\Omega u^2\Delta v = -\frac{1}{d}\int_\Omega u^2(v-u) \le \frac{1}{d}\int_\Omega u^3 \le \frac{K}{d}\|u\|_{H^1}^{\frac{N}{2}}\|u\|_{L^2}^{3-\frac{N}{2}}. \quad (9)$$

By Young's inequality, we obtain

$$\frac{1}{2}\|u\|_{H^1}^2 \le \frac{1}{\mathcal{D}}\left\{\|u\|_{L^1}+\|u\|_{L^2}^2\right\}+\frac{1}{2}\|u\|_{L^2}^2+C\|u\|_{L^2}^{\frac{2(6-N)}{4-N}}. \quad (10)$$

Then it holds that for $N<4$,

$$\|u\|_{H^1}^2 \le 2\left(\frac{1}{\mathcal{D}}+C\right)\left(\|u\|_{L^1}+\|u\|_{L^2}^2+\|u\|_{L^2}^{\frac{2(6-N)}{4-N}}\right). \quad (11)$$

Therefore, $u\in L^6(\Omega)$ and $v\in W^{1,6}(\Omega)$. By using $v\in W^{2,6}(\Omega)\subset C^1(\overline{\Omega})$ and the elliptic regularity theory, we obtain $u\in H^2(\Omega)\subset W^{1,6}(\Omega)\cap C^0(\overline{\Omega})$ and $u\in W^{2,6}(\Omega)\subset C^1(\overline{\Omega})$. ∎

Theorem 2.3 *For any positive sequence $\{\mathcal{D}_n\}$ with $\lim_{n\to\infty}\mathcal{D}_n=\infty$, let $(u_n,\ v_n)$ be any sequence of solutions of* (2) *with $\mathcal{D}=\mathcal{D}_n$. Then there exists a solution $(u_\infty,\ v_\infty)$ of* (3), (4)

$$\lim_{n\to\infty}(u_n,\ v_n)=(u_\infty,\ v_\infty) \quad in \quad C(\overline{\Omega})\times C(\overline{\Omega}) \quad (12)$$

passing to a subsequence.

Proof. It follows from Lemma 2.2 that $\{(u_n,\ v_n)\}$ is uniformly bounded in $W^{2,6}(\Omega)\times W^{2,6}(\Omega)$ with respect to $\mathcal{D}_n$. By using Sobolev's theorem, we note $W^{2,6}(\Omega)\subset C^1(\overline{\Omega})$ with compact embedding (see [8]). Then we find a subsequence $\{(u_{n'},\ v_{n'})\}$ and $(u_\infty,\ v_\infty)\in W^{2,6}(\Omega)\times W^{2,6}(\Omega)$ such that

$$\lim_{n'\to\infty}(u_{n'},\ v_{n'})=(u_\infty,\ v_\infty) \quad (13)$$

weakly in $W^{2,6}(\Omega)\times W^{2,6}(\Omega)$ and strongly in $C^1(\overline{\Omega})\times C^1(\overline{\Omega})$. The weak forms of (2) with $\mathcal{D}_{n'}$ can be expressed by

$$\begin{cases} \displaystyle\int_\Omega(\nabla u_{n'}-\alpha u_{n'}\nabla v_{n'})\nabla\varphi\ dx=\frac{1}{\mathcal{D}_{n'}}\int_\Omega f(u_{n'})\varphi\ dx, \\ \displaystyle d\int_\Omega \nabla v_{n'}\nabla\varphi\ dx=\int_\Omega(u_{n'}-v_{n'})\varphi\ dx \end{cases} \quad (14)$$

for any $\varphi\in H^1(\Omega)$. By virtue of (13), letting $n'\to\infty$ in (14) gives the fact that $(u_\infty,\ v_\infty)$ is a weak solution of (3). It follows from the elliptic regularity theory that $(u_\infty,\ v_\infty)$ becomes a classical solution of (3). Furthermore, integrating the first equation of (2) with $\mathcal{D}_{n'}$, one can see

$$\int_\Omega f(u_{n'})\ dx=0. \quad (15)$$

Owing to (13), the Lebesgue dominated convergence theorem enables us to let $n'\to\infty$ in (15) to obtain (4) with $(u,\ v)=(u_\infty,\ v_\infty)$. Therefore, we know that (u_∞,v_∞) is a solution of the shadow system (3), (4). ∎

3 Global bifurcation structure of solutions of (3), (4) as $N = 1$

First, we remark from (3) that u is represented by $u = Ee^{\alpha v}$ for any positive constant E. Then (3), (4) are rewritten as

$$\begin{cases} dv_{xx} + g(v, E) = 0, & x \in (0,1), \\ v(x) \geq 0, & x \in (0,1), \\ v_x(0) = v_x(1) = 0, & \end{cases} \tag{16}$$

and

$$\int_0^1 f(Ee^{\alpha v})\, dx = 0, \tag{17}$$

where $g(v, E) = Ee^{\alpha v} - v$.

Although the global structure of solutions of (16) for a parameter d is already known, we need some estimates to solve the integral constraint (17). Here, we only treat a monotone increasing solution $v(x,\ d,\ E)$ of (16), (17) because all oscillating and reflecting solution can be constructed by connecting rescaling parts of monotone solutions.

By using the bifurcation theory, the solutions of (16) is obtained as one bifurcated from the large constant solution $v^*(E)$ at the bifurcation point $d = d^*(E) = (\alpha v^*(E) - 1)/\pi^2$ (e. g., [5], [27], [28], [30]).

Theorem 3.1 *For any $E \in (0,\ 1/\alpha e)$ and $d \in (0,\ d^*(E))$, there exists a nonconstant solution $v(x,\ d,\ E)$ of* (16) *which satisfies $v_*(E) < v(x,\ d,\ E)$, $v^*(E) \leq \max_{x\in[0,\ 1]} v(x,\ d,\ E)$ and there is $\eta(E) > v^*(E)$ such that*

$$\lim_{d\to 0} v(x,\ d,\ E) = \begin{cases} v_*(E) & \textit{compact uniformly in } [0,\ 1), \\ \eta(E) & x = 1, \end{cases} \tag{18}$$

$$\lim_{d\to d^*(E)} v(x,\ d,\ E) = v^*(E) \quad \textit{uniformly in } [0,1]$$

where $\displaystyle\int_{v_*(E)}^{\eta(E)} g(v,\ E)\ dv = 0.$

Our goal is to derive the global structure of solutions of (16) satisfying the integral constraint (17) for two parameters d, E.

Theorem 3.2 [32] *For any $E \in (0,\ e^{-\alpha})$,* (16), (17) *admits at least one nonconstant solution $v(x,\ d,\ E)$ for some $d = d(E) \in (0,\ d^*(E))$. Moreover, there exists a sequence $\{v(\cdot,\ d_n,\ E_n)\}_{n=1}^{\infty}$ of solutions of* (16), (17) *such that*

$$\lim_{n\to\infty} (d_n,\ E_n) = (0,\ 0) \tag{19}$$

and

$$\lim_{n\to\infty} v(x,\ d_n,\ E_n) = \begin{cases} 0 & \textit{compact uniformly in } [0,\ 1), \\ \infty & x = 1. \end{cases} \tag{20}$$

(i) *If $0 < \alpha < 1$, there is no nonconstant solution of* (16), (17) *for $E \in (e^{-\alpha},\ \infty)$. Moreover, in a neighborhood of the singular limit $(d,\ E) = (0,\ e^{-\alpha})$, all nonconstant solution of* (16), (17) *can be expressed by a local curve $\{(v(\cdot,\ d,\ E(d)) \mid 0 < d < \delta_1\}$, where $E(d)$ is a smooth function and δ_1 is some small number.*

(ii) *If $\alpha > 1$, in a neighborhood of the bifurcation point $(d,\ E) = (d^*(e^{-\alpha}),\ e^{-\alpha})$, all nonconstant solution of* (16), (17) *can be expressed by a local curve $\{(v(\cdot,\ d,\ E(d)) \mid 0 \leq d^*(e^{-\alpha}) - d < \delta_2\}$, where $E(d)$ is a smooth functions and δ_2 is some small number.*

Sketch of Proof: In order to prove the theorem, let T be a domain defined by $T := \{(E,\ d) \mid 0 < E \le 1/\alpha e,\ 0 < d \le d^*(E)\}$. By using Theorem 2.7 in [30], one can verify that the bifurcation at $d = d^*(E)$ is subcritical with respect to d because that $g(v,\ E)$ is an A-B-function. Therefore there is no nonconstant solution of (16) for $(E,\ d) \in \mathbf{R}^2_+ \setminus T$ from Theorem 3.1.

To obtain solutions of (16) satisfying the integral constraint (17), we may only consider for $(E,\ d) \in T$. Setting

$$\Phi(d, E) = \int_0^1 f(Ee^{\alpha v})\ dx = \int_0^1 Ee^{\alpha v}(1 - Ee^{\alpha v})\ dx$$

for the solution $v(x,\ d,\ E)$ of (16), we will obtain solutions of (16) satisfying $\Phi(d,\ E) = 0$.

First we consider the value of $\Phi(d, E)$ on the boundary of the domain T except $E = 0$. Since $\lim_{d \to d^*(E)} v(x, d, E) = v^*(E)$ in $C^0([0,1])$, we can define $\Phi^*(E) := \lim_{d \to d^*(E)} \Phi(d,\ E) = f(Ee^{\alpha v^*(E)})$. Moreover, it follows from Theorem 3.1 and Lebesgue convergence theorem that

$$\Phi_*(E) := \lim_{d \to 0} \Phi(d, E) = f(Ee^{\alpha v_*(E)}). \tag{21}$$

Therefore, we will show the signs of $1 - Ee^{\alpha v_*(E)}$ and $1 - Ee^{\alpha v^*(E)}$ because of $f(u) = u(1-u)$ and $u = Ee^{\alpha v}$.

To do so, we introduce two functions $\Psi_*(E)$, $\Psi^*(E)$ by $\Psi_*(E) = Ee^{\alpha v_*(E)}$ and $\Psi^*(E) = Ee^{\alpha v^*(E)}$. Then we can prove that $\Psi_*(E)$ and $\Psi^*(E)$ are monotone increasing and decreasing for $E \in (0,\ 1/\alpha e)$ such that $\Psi_*(1/\alpha e) = \Psi^*(1/\alpha e) = 1/\alpha$, $\Psi_*(0) = 0$ and $\lim_{E \to 0} \Psi^*(E) = \infty$.

First, we assume $\alpha < 1$. Since $\Psi_*(E) < 1/\alpha$ for $E \in (0,\ 1/\alpha e)$, there is only $E_* \in (0, 1/\alpha e)$ such that $\Psi_*(E_*) - 1 = E_* e^{\alpha v_*(E_*)} - 1 = 0$. Therefore, we have $v_*(E_*) = 1$ and $E_* = e^{-\alpha}$ because of $g(v_*(E_*),\ E_*) = E_* e^{\alpha v_*(E_*)} - v_*(E_*) = 0$. Moreover, it holds that $0 < \Psi_*(E) < 1$ for $E \in (0,\ e^{-\alpha})$ and $\Psi_*(E) > 1$ for $E \in (e^{-\alpha},\ 1/\alpha e)$.

Since $\Phi(d^*(E),\ E) < 0$ for any $0 < E < 1/\alpha e$ and $\lim_{d \to 0} \Phi(d,\ E) = \Psi_*(E)(1 - \Psi_*(E)) > 0$ for any $0 < E < e^{-\alpha} < 1/\alpha e$, there exist solutions of (16), (17) with some $0 < d(E) < d^*(E)$ for any $0 < E < e^{-\alpha}$ by the intermediate value theorem.

Moreover, since $\lim_{E \to E_*} \frac{\partial}{\partial E} \Phi_*(E) = -E_* e^{2\alpha} < 0$ (see [18]), it holds that there exists an unique solution $v(x,\ d(E),\ E)$ in the neighborhood of $(E,\ d) = (E_*,\ 0)$ with respect to d by the implicit function theorem.

Next, we show the nonexistence of the nonconstant solution of (16), (17) for $(E,\ d) \in T$ and $E \in (e^{-\alpha},\ 1/\alpha e)$. Thanks to Theorem 3.1, we have $1 = v_*(e^{-\alpha}) < v_*(E) < v(x, d, E)$ for any $d \in (0,\ d^*(E_*))$. Therefore, it holds that

$$\Phi(d, E) < E \int_0^1 e^{\alpha v(x,d,E)}(1 - e^{\alpha(v(x,d,E)-1)})\ dx < 0.$$

Therefore, there is not any point $(E, d) \in T$ satisfying $\Phi(d, E) = 0$.

Next, we consider the case (ii), that is, $\alpha > 1$. By using a similar argument as the above, we show that $\Phi^*(E) < 0$ for $E \in (0, e^{-\alpha})$ and $\Phi^*(E) > 0$ for $E \in (e^{-\alpha}, 1/\alpha e)$. On the other hand, $\Phi_*(E)$ satisfies $\Phi_*(E) > 0$ for any $E \in (0, 1/\alpha e)$. Then the solutions $v(x,\ d(E),\ E)$ of (16), (17) with some $0 < d(E) < d^*(E)$ are obtained for any $E \in (0, 1/\alpha e)$ by the intermediate value theorem.

Since $\lim_{E \to e^{-\alpha}} \frac{\partial}{\partial E} \Phi(d^*(E),\ E) = \frac{\alpha e^{-\alpha}}{\alpha - 1} e^{\alpha v^*(e^{-\alpha})} > 0$, there exists an unique solution $v(x,\ d(E),\ E)$ in the neighborhood of $(E,\ d) = (e^{-\alpha},\ d^*(e^{-\alpha}))$ with respect to d from the implicit function theorem. ∎

4 Sensitive function of Keller-Segel type

In this section, we consider the another type of the sensitive function

$$\chi(v) = \log v.$$

Then, (2) is rewritten as

$$\begin{cases} 0 = \mathcal{D}\nabla(\nabla u - \alpha u \nabla \log v) + f(u), & x \in \Omega, \\ 0 = d\Delta v + u - v, & x \in \Omega, \\ u \geq 0,\ v \geq 0, & x \in \Omega, \\ u_\nu = v_\nu = 0, & x \in \partial\Omega. \end{cases} \tag{22}$$

As $\mathcal{D} \to \infty$, it follows from the first equation of (22) and the boundary conditions that

$$\nabla u - \alpha u \nabla \log v = 0. \tag{23}$$

Therefore, u is given by

$$u = Ev^\alpha \tag{24}$$

with a positive constant E.

Then, (22) is rewritten as

$$\begin{cases} 0 = d\Delta v + Ev^\alpha - v, & x \in \Omega, \\ v \geq 0, & x \in \Omega, \\ v_\nu = 0, & x \in \partial\Omega \end{cases} \tag{25}$$

with the integral constraint

$$\int_\Omega f(Ev^\alpha)\, dx = 0. \tag{26}$$

By using $v = E^\alpha w$, we have

$$\begin{cases} 0 = d\Delta w + w^\alpha - w, & x \in \Omega, \\ w \geq 0, & x \in \Omega, \\ w_\nu = 0, & x \in \partial\Omega \end{cases} \tag{27}$$

with the integral constraint

$$\int_\Omega f(E^\delta w^\alpha)\, dx = 0 \tag{28}$$

for $\delta = 1/(1-\alpha)$.

Therefore, (27) does not include the parameter E and is studied by many people (see subcritical case [9], [22], [24], [33], [34], supercritical [23]).

Hereafter, we assume

$$\alpha \neq 1. \tag{29}$$

It follows from (28) that E is given by

$$E = \left(\int_\Omega w^\alpha\, dx \Big/ \int_\Omega w^{2\alpha}\, dx \right)^{1/\delta}. \tag{30}$$

Since the constant E is determined by the solution w of (27), we only consider the problem (27).

Hereafter, we treat the 1-dim. problem corresponding to (25), (28). Let $\Omega = (0,1)$. Then, this problem is rewritten as

$$\begin{cases} 0 = dw_{xx} + w^{\alpha} - w, & x \in (0,1), \\ w(x) \geq 0, & x \in (0,1), \\ w_x(0) = w_x(1) = 0 \end{cases} \tag{31}$$

and

$$E = \left(\int_0^1 w^{\alpha}\, dx \Big/ \int_0^1 w^{2\alpha}\, dx \right)^{1/\delta}. \tag{32}$$

Letting $w^* = 1$ and $d^* = (\alpha-1)/\pi^2$, w^* and d^* are a constant solution of (31) and the bifurcation point from the constant solution, respectively. By using the bifurcation theory, the following theorem was proved.

Theorem 4.1 [35] *There exists a continuous curve* $\{(w(x,d(s)),d(s)) \mid s \in (0,1)\}$ *such that* $w(x,d(s))$ *is a solution of* (31) *with* $d = d(s)$ *where* $\lim_{s\to 0} d(s) = 0, \lim_{s\to 1} d(s) = d^*$ *and*

$$\lim_{s\to 0} w(x,d(s)) = \begin{cases} 0 & \textit{compact uniformly in } [0,1) \\ \hat{w} & x = 1, \end{cases}$$
$$\lim_{s\to 1} w(x,d(s)) = w^* \quad \textit{uniformly in } [0,1]. \tag{33}$$

Here $\hat{w}$ *is some constant given by* $\int_0^{\hat{w}} (w^{\alpha} - w)\, dw = 0$. *Moreover, the continuous function* $d(s)$ *is monotone increasing for* $s \in [0,1]$.

Let $\Phi(s)$ be given by

$$\Phi(s) = \int_0^1 w^{\alpha}\, dx \Big/ \int_0^1 w^{2\alpha}\, dx. \tag{34}$$

Then we have

$$\lim_{s\to 0} \Phi(s) = \Phi(0) = \int_{-\infty}^0 W^{\alpha}(z)\, dz \Big/ \int_{-\infty}^0 W^{2\alpha}(z)\, dz \quad \text{and} \quad \lim_{s\to 1} \Phi(s) = \Phi(1) = 1. \tag{35}$$

Here $W(z)$ is a monotone increasing solution of

$$\begin{cases} 0 = W_{zz} + W^{\alpha} - W, \ -\infty < z < 0, \\ W_z(0) = 0. \end{cases} \tag{36}$$

From Theorem 4.1 and (32), we have

Theorem 4.2 *For any fixed* $\alpha > 1$, *there exists a continuous function* $d(s)$ *and solution* $w(x,d(s))$ *of* (31) *with* $d = d(s)$ $(0 < d(s) < d^*)$ *for* $0 < s \leq 1$. *Moreover, it holds that*

$$\lim_{s\to 0} E(s) = \Phi(0)^{1-\alpha}, \quad \lim_{s\to 1} E(s) = 1. \tag{37}$$

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